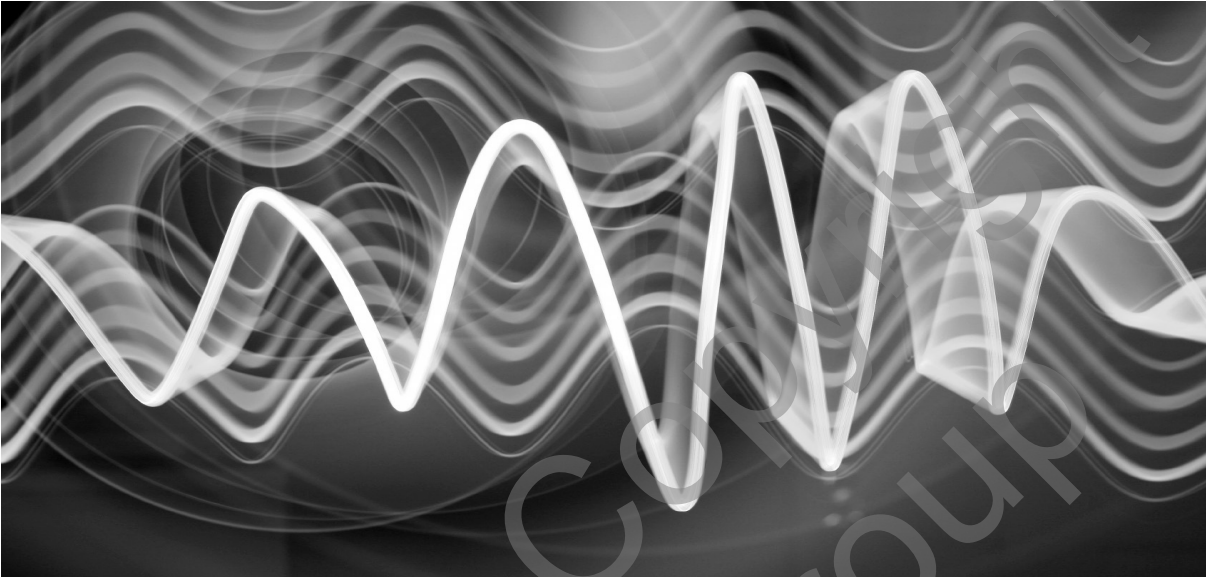


WAVES



Syllabus Checklist

SCIENCE UNDERSTANDING – WAVES

- waves are periodic oscillations that transfer energy from one point to another.
- mechanical waves transfer energy through a medium; longitudinal and transverse waves are distinguished by the relationship between the directions of oscillation of particles relative to the direction of the wave velocity.
- waves may be represented by displacement/time and displacement/distance wave diagrams and described in terms of relationships between measurable quantities, including period, amplitude, wavelength, frequency and velocity. This includes applying the relationships:

$$v = f\lambda$$

$$T = \frac{1}{f}$$

- the mechanical wave model can be used to explain phenomena related to reflection and refraction, including echoes and seismic phenomena.
- the superposition of waves in a medium may lead to the formation of standing waves and interference phenomena, including standing waves in pipes and on stretched strings.
- This includes applying the relationships for:
 - strings attached at both ends and pipe open at both ends: $\lambda = \frac{2l}{n}$
 - pipe closed at one end: $\lambda = \frac{4l}{(2n - 1)}$
- a mechanical system resonates when it is driven at one of its natural frequencies of

oscillation; energy is transferred efficiently into systems under these conditions.

- the intensity of a wave decreases in an inverse square relationship with distance from a point source.

This includes applying the relationship: $I \propto \frac{1}{r^2}$

5.1 NATURE OF WAVES

What is a wave?

Some common examples of waves that are part of our everyday life include water waves, sound waves, microwaves, radio waves and light waves. The most important feature of these waves (as with all waves) is that they carry energy and quite often information from one point to another.

Wave motion may be simply defined as a disturbance or vibration which transmits energy without the transport of matter. Mechanical waves, such as sound, are those that travel through a medium due to the vibration of the particles of the medium. Electromagnetic waves, such as light, do not need a medium to travel through. A periodic disturbance at a source will create a progressive wave as the disturbance spreads from the source. A pulse is a single wave or disturbance.

Types of waves

Waves are classified according to the motion of the particles of the medium transmitting them. There are two types - transverse and longitudinal waves.

Transverse waves: Those in which each particle vibrates in a direction perpendicular to that of the energy flow. Examples are light, radio and water waves. Waves set up in a string as shown below are transverse waves.

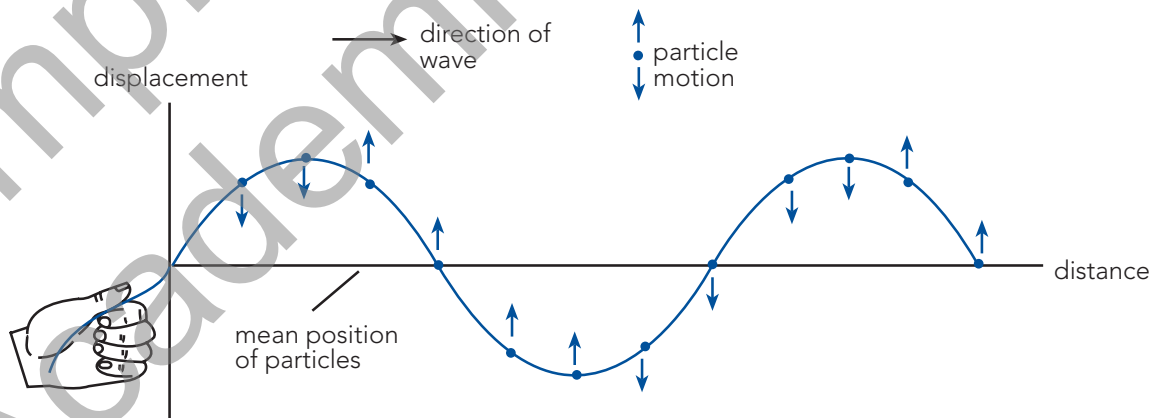


Figure 5.1 Transverse wave motion such as waves in a string. Each point on the wave vibrates at right angles to the direction of the wave.

Longitudinal waves: Those in which the particles vibrate along straight lines parallel to the direction of the energy flow. Example: sound waves.

Longitudinal waves can be set up in a slinky spring as shown below. They can also be created by a vibrating source, such as a loudspeaker. The loudspeaker will cause air next to it to be alternatively pushed away (compression) and drawn back (rarefaction). This sets up a series of pressure changes in the air which can be represented as shown below.

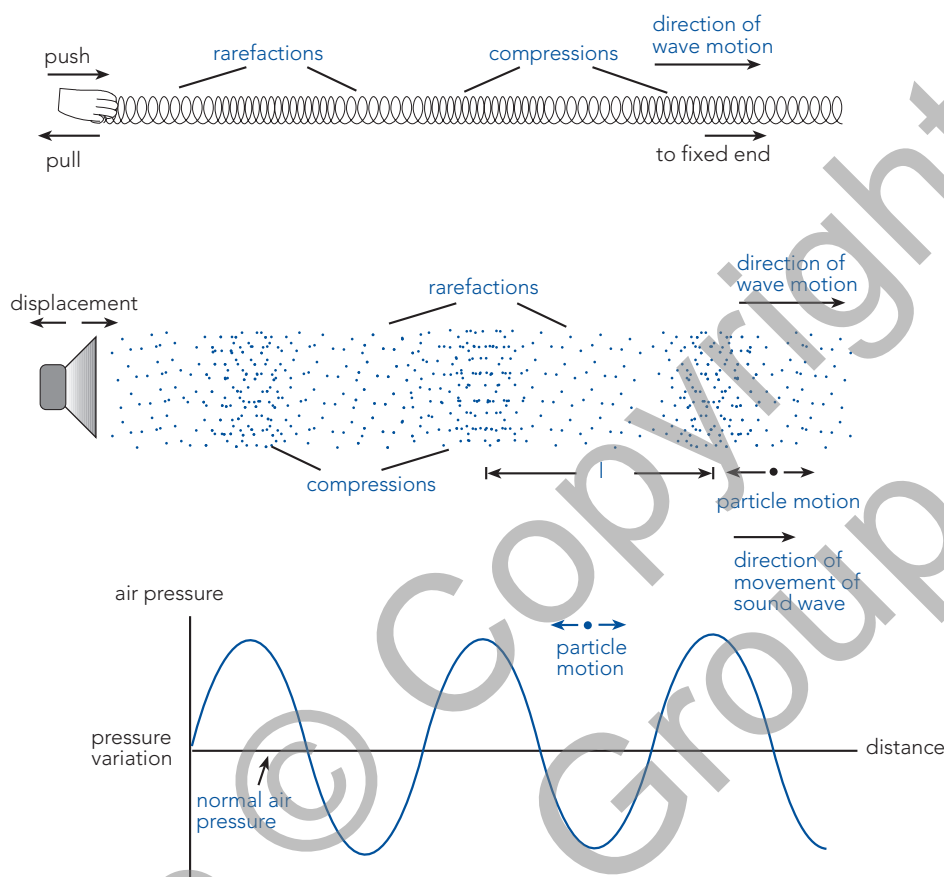


Figure 5.2 Longitudinal wave motion in a slinky spring and in air. Each particle in the wave vibrates back and forth parallel to the direction of the wave. The wave motion can also be represented graphically as shown.

Wave graphs

Wave motion can be represented by a displacement/time graph or a displacement/distance graph.

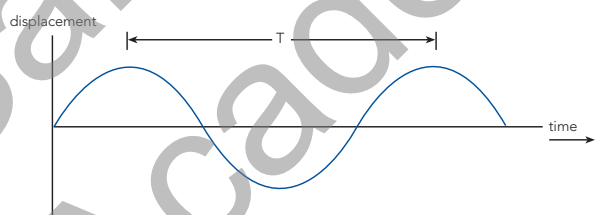


Figure 5.3 Displacement/time graph

This graph represents the motion of a single particle as a wave passes through it. The crests and troughs represent the particle's maximum displacement from its mean position and the points at which it has zero velocity.

T is the period of vibration.

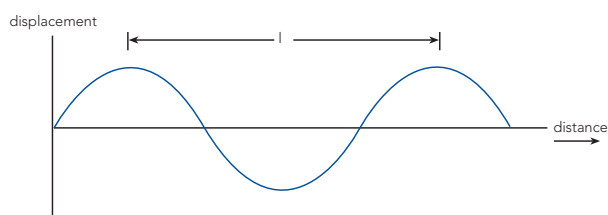


Figure 5.4 Displacement/distance graph

This graph represents the displacement of a number of particles from their mean position at some instant in time. We can think of it as a "snapshot" which shows how the displacement of the medium varies over the distance from the source.

λ represents the wavelength of the wave.

Describing wave motion – Important terms

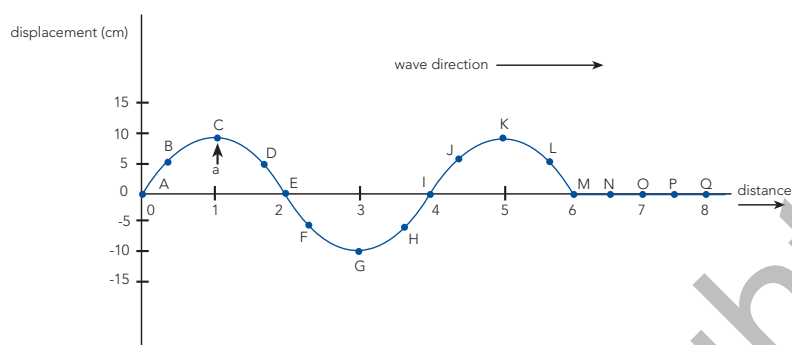


Figure 5.5 Describing the motion of a wave. Several points on the transverse wave shown above are labelled so as to help define the terms used to describe wave characteristics.

Phase: The stage of vibration that a particle has reached in its cycle of motion. Particles are in phase if they are moving in exactly the same manner at the same time. In the wave shown above for example, particles A & I or D & L are in phase, while particles C & G are 180° out of phase.

Wavelength (λ): The distance between any two consecutive points in a wave which are in phase. For example, the distance between points C & K, D & L, etc.

Amplitude (a): The maximum displacement of a particle from its mean position (i.e. height of a crest or depth of a trough). The amplitude of the wave shown above is 10 cm.

Frequency (f): The number of complete vibrations (or waves) per second. The unit of frequency is the hertz (Hz).

Period (T): The time taken to complete one vibration (or to produce one complete wave).

Note: $T = \frac{1}{f}$.

Velocity (v): The speed with which a progressive wave travels through a medium. (See wave equation next page).

Special Note: Individual particle motion must not be confused with wave velocity through a medium. Different particles are moving with different velocities at any given instant. In the wave shown above:

- D is moving upwards while B is moving downwards.
- C has an instantaneous velocity of zero.
- E is undergoing maximum velocity (upwards).

Question 5.1

Complete the following by referring to Figure 5.5. Give units where appropriate.

- (a) The wavelength (λ) is _____
- (b) The amplitude (a) is _____
- (c) A point in phase with A is _____
- (d) A point totally out of phase with A is _____
- (e) Three points with zero velocity are _____

- (f) A point undergoing maximum upward velocity is _____
- (g) If the velocity of this wave is 8.0 ms^{-1} , how long will it be before point Q begins to move? _____
- (h) If the frequency (f) of this wave is 2.0 Hz , what is its period (T)?

Wave velocity - The wave equation

For any given wave, its velocity, frequency and wavelength are related as shown by the wave equation:

$$v = f \lambda$$

v = wave velocity (ms^{-1})

f = frequency (Hz)

λ = wave length (m)

Velocity (and wavelength) are affected by the medium that a wave is travelling through whereas the frequency is dependent only on its source.

The speed of sound waves in various substances is shown in Table 5.1 below. Where the sound originates from the same source (say a tuning fork of frequency 512 Hz) the frequency remains constant no matter which medium it travels through. The velocity will change however, as will the wavelength by the same proportion.

Table 5.1 Speed of Sound (in selected substances)

Substance	Velocity (ms^{-1})	Substance	Velocity (ms^{-1})
Gases (25°C , 101.3 kPa)		Solids (thin rods)	
air, dry	3.46×10^2	aluminium	5.00×10^3
carbon dioxide	2.69×10^2	brass	3.48×10^3
helium	9.85×10^2	brick	3.65×10^3
nitrogen	3.49×10^2	copper	3.81×10^3
oxygen	3.30×10^2	iron (soft)	5.20×10^3
Liquids (25°C)		cork	5.00×10^2
glycerol	1.90×10^3	glass (crown)	4.54×10^3
benzene	1.31×10^3	iron (cast)	4.48×10^3
water, distilled	1.50×10^3	lead	1.20×10^3
water, sea	1.53×10^3	silver	2.68×10^3

Worked Example 5.1

One of the loudspeakers from a sound system is capable of producing good quality sound in the frequency range of 450 – 4000 Hz. Assuming the speed of sound to be 340 ms^{-1} find:

- (a) the maximum distance between compression waves emitted by this speaker;
- (b) the smallest time interval between successive compression waves.

$$\begin{aligned}v &= 340 \text{ ms}^{-1} & \lambda_{(\text{max})} &= ? \\f_{(\text{min})} &= 450 \text{ Hz} & T_{(\text{min})} &= ? \\f_{(\text{max})} &= 4000 \text{ Hz}\end{aligned}$$

- (a) Since velocity is constant, the maximum wavelength will occur with the lowest frequency.

$$\begin{aligned}v &= f\lambda \\ \therefore \lambda &= \frac{v}{f} \\ &= \frac{340}{450} \\ &= 0.756 \text{ m}\end{aligned}$$

\therefore Max. distance between compressions will be 0.756 m.

- (b) Smallest period (T) will be given by the highest frequency.

$$\begin{aligned}T &= \frac{1}{f} = \frac{1}{4000} = 2.50 \times 10^{-4} \text{ s} \\ \therefore \text{Smallest time interval} &= 2.50 \times 10^{-4} \text{ s}\end{aligned}$$

Question 5.2

Assuming that it was possible to transmit the sound from the speaker in example 1 through sea water, calculate (using data from Table 5.1):

- (a) The maximum wavelength of the sound.

- (b) The time it would take the sound to reach a point in the sea water 1.00 km away.

Question 5.3

Ripples are generated in a pond when a small stone is dropped into it. After 4.00 seconds it was noticed that there were 24 ripples created and the furthest ripple was 2.00 m from the spot the stone was dropped. Determine the frequency, wavelength and velocity of the wave.

Intensity of a wave

As we have seen, waves transfer energy from one point to another. Sound waves, for example, transmit the energy produced by a source, such as a loudspeaker, in all directions. The sound travels away from the source as a spherical wave, and its intensity decreases rapidly with distance.

The intensity of a sound wave is defined as the amount of energy passing per second through an area of one square metre.

ie. Intensity = $\frac{\text{energy}}{\text{time} \times \text{area}}$

Also, since Power = $\frac{\text{energy}}{\text{time}}$

We have

$$I = \frac{P}{A}$$

I = Intensity (Wm^{-2})

P = Power (W)

A = Area (m^2)

How intensity varies with distance

As we move away from a sound source, the sound energy is spread over a larger area and hence the sound intensity falls. The inverse square law applies as shown below.

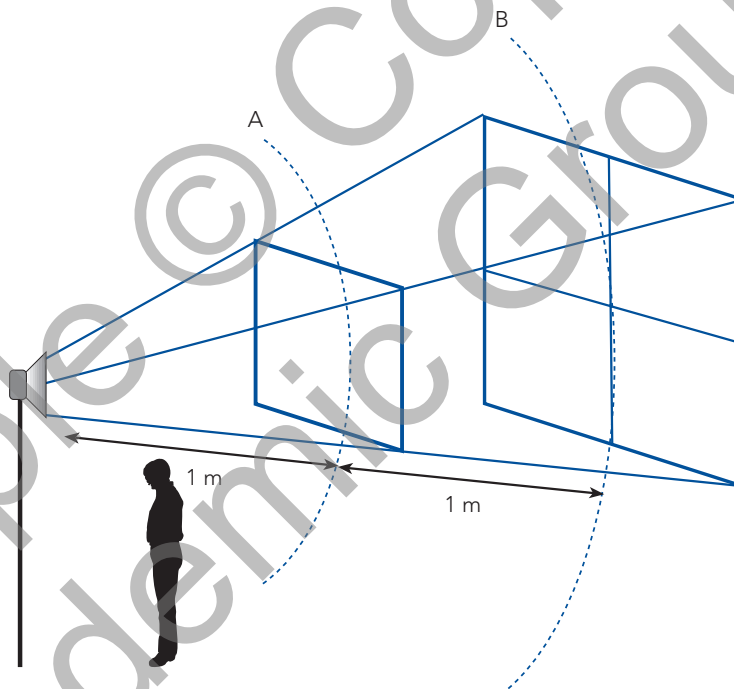


Figure 5.6 The intensity of a sound wave decreases in an inverse square relationship with distance from a point source. For example, at position B as shown above, the intensity of a sound wave will be $\frac{1}{4}$ of that at position A.

Sound travels from a source as a spherical wave. The intensity at a distance r from a point source is given by:

$$I \propto \frac{1}{r^2}$$

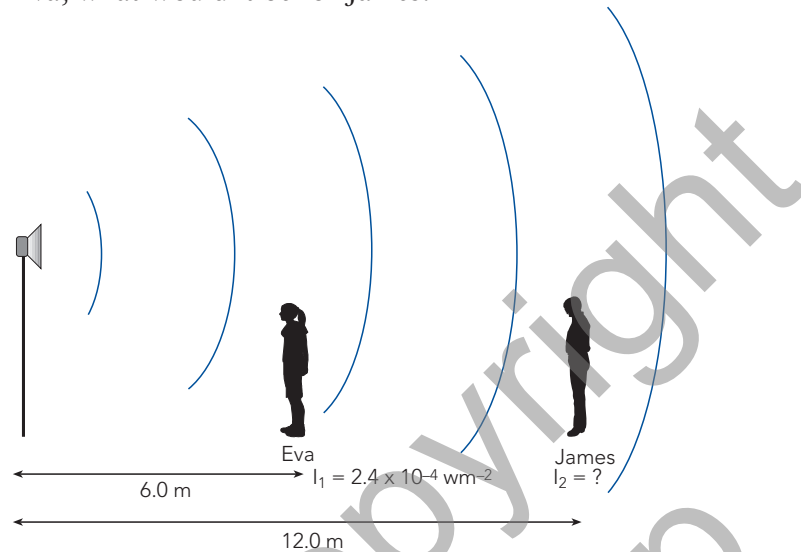
I = Intensity (Wm^{-2})

r = Distance from point source

This means, for example, that if the distance from the source doubles, the intensity will fall to $\frac{1}{4}$ of its original value. We can see from the diagram above that this is due to the fact that the sound energy is spread over 4 times the original area.

Worked Example 5.2

Eva and James are standing 6.0 m and 12.0 m respectively from a loudspeaker. If the intensity is $2.4 \times 10^{-4} \text{ Wm}^{-2}$ for Eva, what would it be for James?



There are two different ways this calculation can be done.

Method 1 Consider the inverse square law.

$$I \propto \frac{1}{d^2} \quad \therefore \quad I d^2 = \text{constant}$$

$$\text{Hence } I_2 d_2^2 = I_1 d_1^2$$

$$I_2 = 6.0 \times 10^{-5} \text{ Wm}^{-2}$$

Method 2 Consider total acoustical power.

$$\begin{aligned} P &= I A \\ \text{where Eva is } P &= (2.4 \times 10^{-4}) (4 \pi \cdot (6.0)^2) \\ &= 0.109 \text{ W} \end{aligned}$$

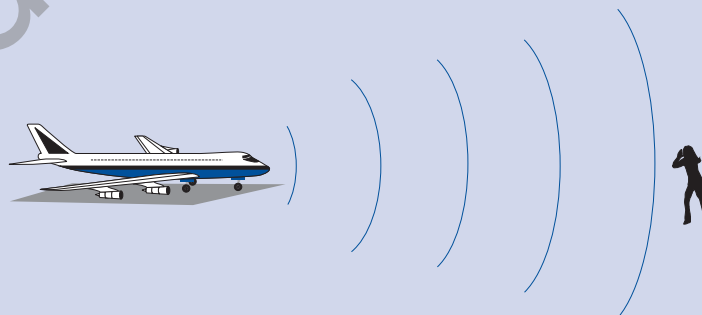
$$\begin{aligned} \text{where James is } P &= I_2 A = 0.109 \\ 0.109 &= (I_2) (4 \pi \cdot (12)^2) \\ I_2 &= 6.0 \times 10^{-5} \text{ Wm}^{-2} \end{aligned}$$

Where A is the surface area of a sphere at the point under consideration.

Note:

- (1) S.A. (sphere) = $4 \pi r^2$
- (2) The acoustical power of the source, P , is constant.

Question 5.4



Paula is standing 200 m from a jet aircraft listening to the sound of the engines. She measures the intensity of the sound at that point and finds it to be 0.150 Wm^{-2}

(a) Determine the total acoustical power of the sound source.

(b) Calculate the likely intensity of sound that Paula would experience if she walked 50 m closer to the aircraft.

(c) **Challenge:** How far from the aircraft would Paula need to be for the intensity to drop to 0.01 Wm^{-2}

Reflection of waves

When waves reach a boundary between two media reflection will occur. Some transmission and absorption may also take place but the extent of this depends on the media involved.

Reflection of waves is most easily understood if we observe the action of water waves when they encounter a solid barrier (see diagrams below). The wavefronts are readily visible and we can see how they reflect and also create interference patterns. We can see for example that the angle of incidence and angle of reflection are equal.

Sound waves are reflected in a similar manner to all waves. They are best reflected by hard smooth surfaces such as tiled floors, walls and ceilings, but tend to be absorbed by soft and rough surfaces.

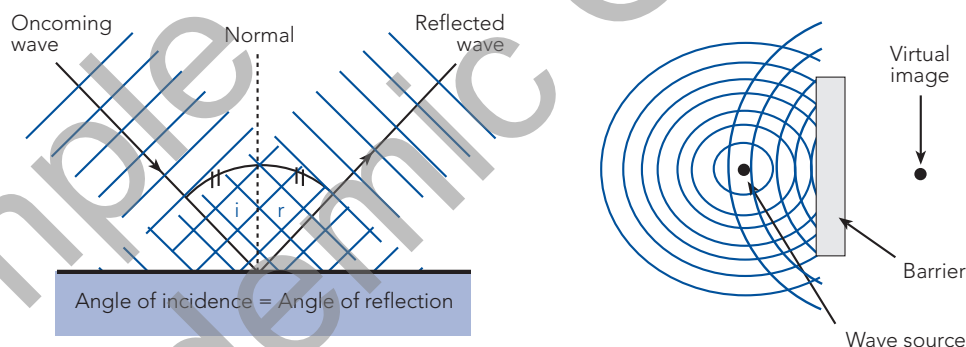


Figure 5.7 The reflection of waves such as water and sound

The reflection of sound waves can cause echoes, makes depth sounding possible and gives rooms their acoustic properties. It also makes possible the operation of doctors' stethoscopes, speaking tubes and brass musical instruments. Echoes are heard when a reflected sound reaches a listener more than $1/10$ of a second after the original sound.

Reflection of ultrasound

Ultrasounds are very high frequency sound waves which can be produced as highly directional beams since little diffraction will occur. There are many situations where the reflection of ultrasounds is very useful.

- Sonar (sound navigation and ranging) has many marine applications such as depth sounding and the location of fish and submerged objects. The time taken for a short pulse to be reflected can be used to calculate the distance to the reflecting surface.

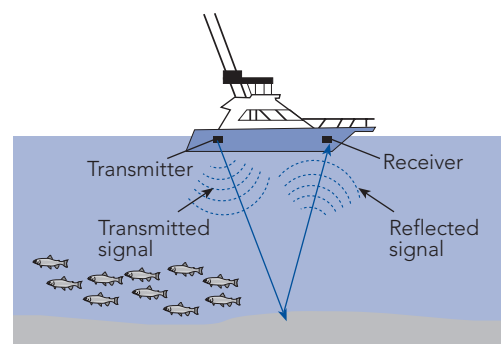


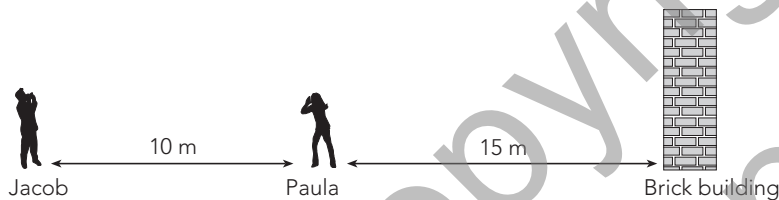
Figure 5.8 Using sonar to locate fish and ocean floor

- Some animals, such as bats and dolphins, use ultrasound to detect objects around them. The very short wavelength of the sounds emitted allows even small objects to be detected.
- In medicine ultrasound waves are used by doctors to obtain pictures of internal parts of the body. Multiple reflections of ultrasound waves can be used to detect tumours, monitor heart function or check the growth of unborn babies.

Worked Example 5.3

Jacob and Paula are standing 25.0 m and 15.0 m respectively from a large building when Jacob calls out loudly. Assume the speed of sound to be 340 ms^{-1} and that good reflection of sound occurs from the building.

- Find the time delay between the two sounds Paula will hear.
- Will Paula detect an echo?
- Will Jacob detect an echo?



$$v = 340 \text{ ms}^{-1}$$

for Jacob

$$d = 50 \text{ m (echo)}$$

$$t = ?$$

for Paula

$$d_1 = 10 \text{ m (direct sound)} \quad t_1 = ?$$

$$d_2 = 40 \text{ m (echo)} \quad t_2 = ?$$

$$(a) \quad \text{Since } v = \frac{s}{t} \quad t = \frac{s}{v}$$

$$\therefore t_1 = \frac{10}{340} = 2.94 \times 10^{-2} \text{ s and } t_2 = \frac{40}{340} = 0.118 \text{ s}$$

$$\Delta t = t_2 - t_1 = 8.82 \times 10^{-2} \text{ s}$$

- The sounds are too close together for Paula to detect an echo.
- For Jacob:

$$t = \frac{50}{340} = 0.147 \text{ s} \quad \text{He will detect an echo.}$$

Question 5.5

In Worked Example 5.3, suppose Paula calls out loudly, would Jacob hear an echo?

Question 5.6

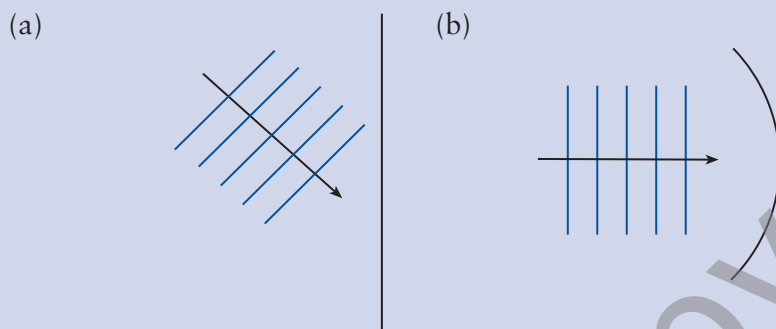
A sonar device uses ultrasound pulses of 50 kHz to map the contours of the bottom of the ocean. If the pulses which are reflected from the bottom reach the ship exactly 0.125 s after being emitted what must be the depth of the ocean at this point? (You will need data from Table 5.1.)

Question 5.7

The following diagrams show sound waves about to be reflected by:

- (a) the corner of a room (b) a circular reflector.

Carefully complete the diagrams to show what would happen in each case.



Refraction of waves

When waves travel from one medium into another of different density, their velocity (and wavelength) will change. Frequency is unaffected. This change in velocity can cause the wave to bend and change direction. This bending is referred to as refraction. The normal laws of refraction apply, that is the waves:

- bend *towards* the normal if they enter a medium where their speed is slower.
- bend *away* from the normal if they enter a medium where their speed is greater

Refraction of water waves can be demonstrated using a ripple tank with different depth sections. The speed of water waves in the shallow sections is less as the wavelength becomes shorter. This causes the wave front to change direction towards the normal.

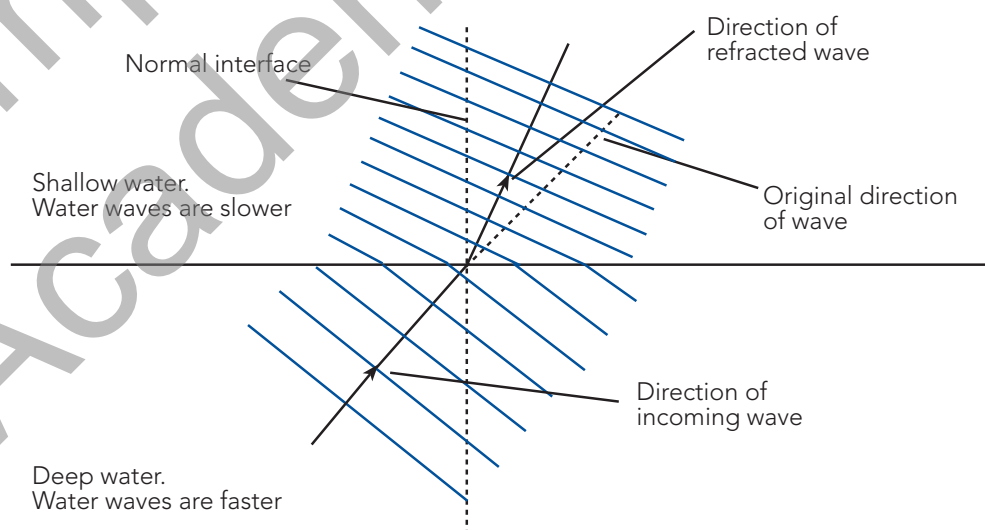


Figure 5.9 Refraction of water waves towards the normal as they progress from deep water to shallow water. Note the decrease in wavelength. Sound waves refract in a similar way

Question 5.8

On a calm summer evening the air near the ground is rather cool while higher up it is warmer. Sound travels faster in warm air. Using wavefront diagrams carefully show how refraction of sound can allow the noise from an aircraft to be heard quite clearly at some point far away.



Diffraction of waves*

Waves, are able to bend around obstacles or around narrow openings placed in their path. The amount of diffraction depends both on wavelength and gap size.

- Larger wavelengths are more easily diffracted than smaller wavelengths.
- Diffraction is greatest when the wavelength is larger than the opening or obstacle.

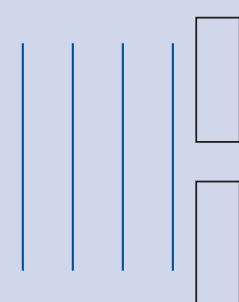
Everyday sound waves are quite easily diffracted as their size (typically some 2 cm to 20 m) is similar to that of most obstacles and openings. This explains why it is quite easy to hear around the corners of a building even though there may be little reflected sound.



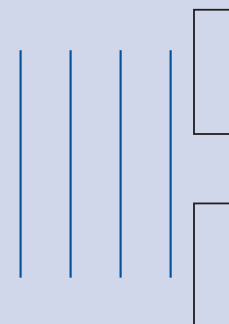
Figure 5.10 Large wavelengths are more easily diffracted

Question 5.9

Complete the wave diagrams below to show how the amount of diffraction of a wave depends on its wavelength compared to that of the opening.



Gap smaller than wavelength



Gap larger than wavelength

Seismic Waves

Seismic waves are caused by events such as earthquakes and travel through the earth's crust in different forms and varying speeds. Seismic waves, like sound, are mechanical waves. They travel through the earth's crust both as longitudinal and transverse waves.

P Waves: The longitudinal waves travel almost twice as fast as the transverse waves and are the first to be felt after a seismic event. They are referred to as *P waves* (primary). Their velocity varies markedly for different materials and increases with depth. Velocity in the Earth's crust can range between 4.0 km/sec to 8.0 km/sec. In granite, for example, it is typically 5.5 km/sec. By comparison the velocity of P waves in liquids is much lower. In water it is some 1.5 km/sec. This large difference in velocities can cause refraction at boundaries.

S Waves: The transverse waves are called *S waves* (secondary). They travel more slowly than P waves and, significantly, can only travel through solids. These waves are second to be felt after a seismic event, have more energy and are generally more destructive. Velocity in the earth's crust can range between 2.5 km/sec to 4.0 km/sec. In granite, for example, it is typically 3.0 km/sec.

Seismic waves can be used to provide a great deal of information about the structure of the Earth. P waves for example are able to travel through solid and liquid materials with some refraction occurring at the boundaries. By analysing the movement, or otherwise, of P and S waves through the earth it is possible to get a better understanding of the earth's layers.

The fact, for instance, that S waves are not able to travel right through the earth indicates the presence of a liquid section near the earth's core. P waves do travel right through the globe but can also be refracted depending on their path. Detailed analysis of wave paths and velocities help to provide a better picture of the earth's structure.

Seismic waves are also very useful in studying the Earth's crust and exploring for resources such as oil. Waves created by artificial sources are reflected by boundaries and discontinuities between rock layers. Refraction can also occur, in particular if liquid areas are encountered. The reflected waves are recorded by a series of geophones and analysed. The seismic cross section that is produced can indicate features such as rock layers, faults, folds and the presence of liquids.

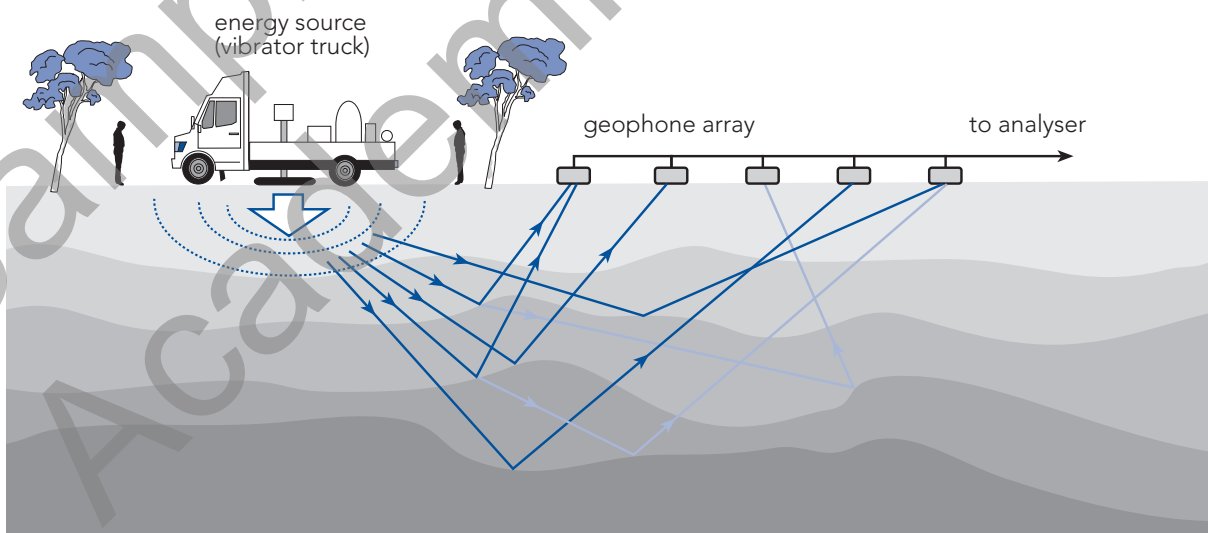


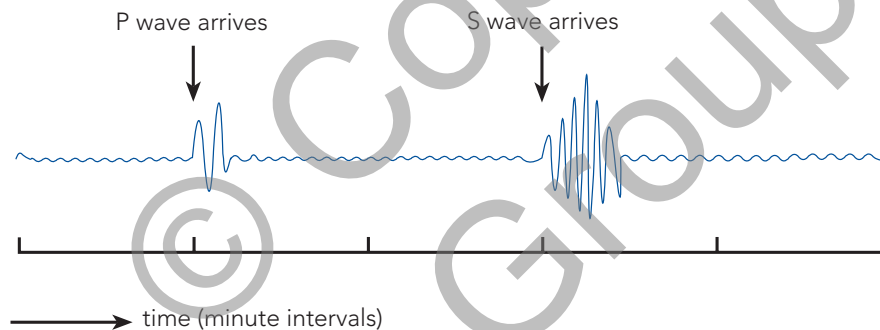
Figure 5.11 Reflection seismology. Seismic waves generated by an energy source such as a vibrator truck are transmitted through soil and rock layers and their reflections recorded and analysed. Sometimes small explosive charges are used. Waves are mostly reflected by layer boundaries or discontinuities. Refraction can also occur. The analysis of the very large number of recorded reflections results in a detailed cross section of the underlying rock layers.

Worked Example 5.4

Earthquakes produce different kinds of waves which travel through the earth's crust at different velocities. This means that they arrive at any particular seismic station at different times. P waves arrive first, followed by S waves. This time difference can be used to directly determine the distance of a seismic event. Analysis of data from many earthquakes gives a value of about 9.0 km for every second of time difference.

However, for the purpose of this worked example, we will use typical average velocities for the P and S waves involved. Assume these to be 5.1 km/sec and 3.2 km/sec respectively. A seismogram is shown below recording the arrival of P and S waves at a recording station. Use this data to answer the following.

- Determine the time difference between the arrival of the two different waves at the recording station.
- Calculate the distance of the seismic event from the recording station using the velocities given.
- The distance is usually determined from established data linking it directly to time difference. Use this direct conversion method to calculate the distance of the seismic event. Comment on any differences and suggest which method may be most useful.



- The time difference = 2.0 minutes (120 s)
- Let unknown distance = s (same for both P and S waves)
 Let time for P wave to arrive = t
 So time for S wave to arrive = $t + 120$
 Since $v = s/t$ we have $s = vt$
 For P waves $s = (5100)(t)$ For S waves $s = (3200)(t + 120)$
 Since s is common we have

$$5100t = 3200t + 384000$$

$$1900t = 384000 \quad t = 202 \text{ s}$$
 Hence distance $s = (5100)(202) = 1030 \text{ km}$
- Conversion value given = $(9.0 \text{ km/s})(\Delta t \text{ between P and S arrival})$
 Hence distance $s = (9.0)(120) = 1080 \text{ km}$

The calculation of distance using a conversion value is certainly the most convenient method. It is also likely to be the most accurate as the conversion factor is based on the analysis of recorded data from many seismic events. In contrast, the velocity of P and S waves can vary widely due to material encountered and path taken.

Adding waves (superposition)

Whenever two waves pass through a point at the same time, interference will occur. However, each individual wave is unaffected and travels through the point as though no other wave existed. The combining of two waves is known as superposition and is simply the algebraic addition of the displacement of the two waves.

- Constructive interference is when two waves combine to give a greater resultant displacement. For similar waves this effect is a maximum when the waves are in phase.
- Destructive interference is when the two waves combine to give a smaller resultant displacement. This effect is a maximum for similar waves 180° out of phase.

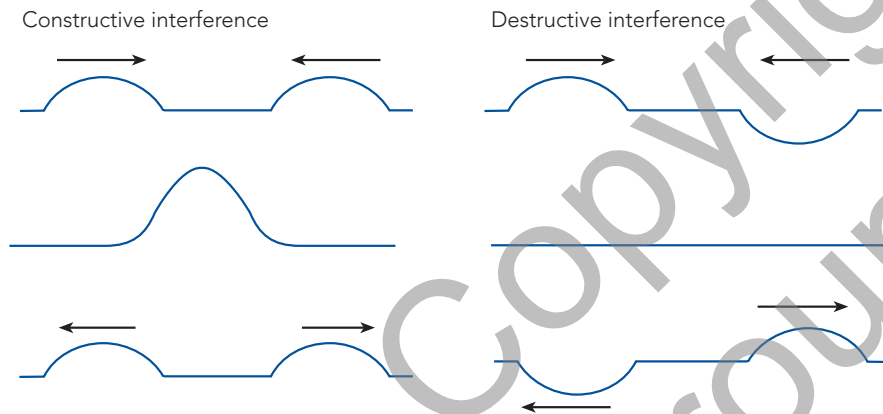
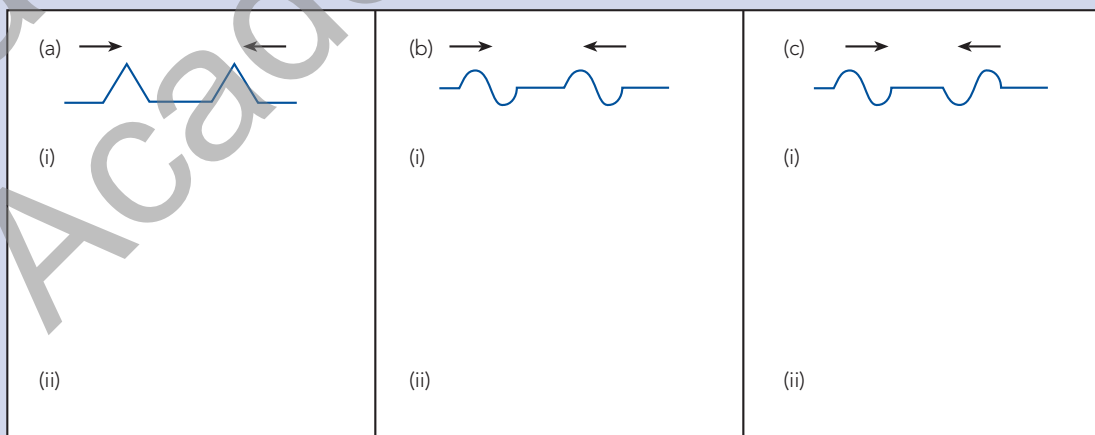


Figure 5.12 Superposition, showing (a) constructive and (b) destructive interference

Question 5.10

Complete the following diagrams to show the resultant displacement when:

- the two pulses coincide totally;
- the two pulses have passed each other.



Interference from two point sources

A very good example of this type of interference occurs when two loudspeakers emit the same sound (in phase) from two different points. Points of maximum reinforcement (ANTINODES) and total annulment (NODES) will occur as shown in the diagram below.

Example A

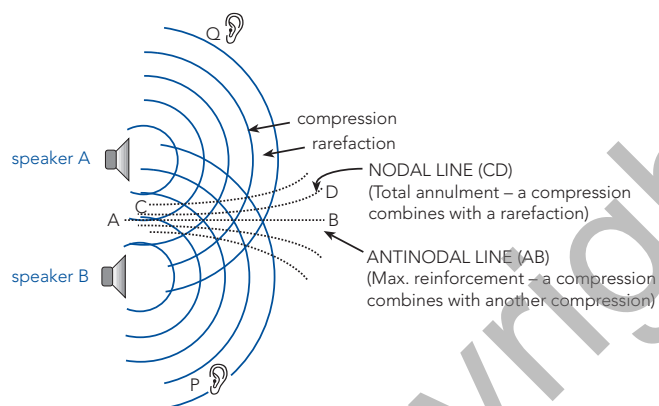


Figure 5.13(a) Interference between two point sources

A similar interference effect (nodes and antinodes) will occur when two loudspeakers are facing each other and emitting the same sound in phase. (In fact, a standing wave pattern forms between the speakers.)

Example B

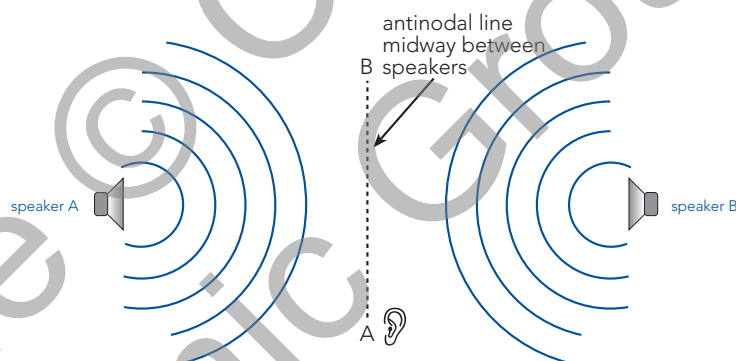


Figure 5.13(b)

Antinodal lines occur where there is constructive interference – a loud sound is heard.

Nodal lines occur where there is destructive interference – a soft sound is heard. (In theory no sound should be heard – why is some sound actually heard?)

Question 5.11

Describe what you would hear, and explain why, if you walked from:

(a) A to B in example A

(b) P to Q in example A

(c) A to B in example B

Worked Example 5.5

Two loudspeakers (A and B) are set up as shown in Figure 5.13(a) so that they both emit sounds of 425 Hz which are in phase. The speakers are 2.40 m apart. Assume the speed of sound is 340 ms^{-1} .

- What is the wavelength of the sounds emitted?
- Melissa stands exactly midway between the speakers and then walks away so as to always remain equidistant from them. Describe what she hears. Explain why.
- Melissa now walks to a point C, 7.00 m directly in front of speaker A.
 - How far is she from speaker B?
 - What will be the effect?

$$\begin{aligned}v &= 340 \text{ ms}^{-1} \\f &= 425 \text{ Hz} \\ \lambda &= ? \\AB &= 2.40 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{(a)} \quad v = \lambda f \quad \therefore \lambda &= \frac{v}{f} = \frac{340}{425} \\ &= 0.800 \text{ m}\end{aligned}$$

(b) If Melissa is always equidistant from the speakers the sounds reaching her will always be in phase. She will therefore always hear a loud sound.

$$\begin{aligned}\text{(c)} \quad AB &= 2.40 \text{ m} & (BC)^2 &= (AC)^2 + (AB)^2 \\ AC &= 7.00 \text{ m} & &= 49 + 5.76 \\ BC &= ? & \therefore BC &= 7.40 \text{ m}\end{aligned}$$

Since the wavelength of the sound is 0.80 m then at point C the sounds are exactly half a wavelength apart (180° out of phase).

Melissa will “hear” silence.

Question 5.12

For the previous question assume Melissa walks parallel to the speakers (as in P to Q in Figure 5.13(a)).

- Describe what she would hear as she walked steadily past the speakers.

- The frequency of the sounds from both speakers is doubled. What will Melissa now hear if she repeats her experiment?
