

## ALGEBRAIC SKILLS

## SYLLABUS CHECKLIST

By the end of this chapter, you should be able to:

- substitute numerical values into algebraic expressions and evaluate
- determine the value of the subject of a formula

## FORMULAE AND DEFINITIONS

## Simplify

To expand brackets, collect 'like' terms and reduce the expression.

## Solve

To find all solutions to an equation, inequality or system of equations.

## Factorise

To remove the highest common factor (HCF) outside a set of brackets.

## Inequalities

These contain the signs  $>$ ,  $<$ ,  $\leq$ , or  $\geq$ .

## WORKED EXAMPLES

2.1 If  $x = 2$ ,  $y = -3$ , and  $z = 4$  calculate:

(a)  $2xy$

(b)  $3yz - x$

(c)  $y^2 + 2x^2z$

(a)  $2xy$   
 $= 2(2)(-3)$   
 $= -12$

(b)  $3yz - x$   
 $= 3(-3)(4) - (2)$   
 $= -36 - 2$   
 $= -38$

(c)  $y^2 + 2x^2z$   
 $= (-3)^2 + 2(2)^2(4)$   
 $= 9 + 2(4)(4)$   
 $= 9 + 32$   
 $= 41$

- 2.2 A pendulum on a clock swings from side to side. The time (T seconds) to complete one cycle (left to right and back to left again) is

$$T = 2\pi\sqrt{\frac{\ell}{g}} \text{ where}$$

$\ell$  = length of the swing (cm)  
 $g$  = gravitational acceleration.

Find T if:

- (a)  $\ell = 6.2, g = 4$   
 (b)  $\ell = 30, g = 8.5$

$$\begin{aligned} \text{(a)} \quad T &= 2\pi\sqrt{\frac{\ell}{g}} \\ &= 2\pi\sqrt{\frac{6.2}{4}} \\ &= 7.82 \text{ seconds} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad T &= 2\pi\sqrt{\frac{30}{8.5}} \\ &= 11.80 \text{ seconds} \end{aligned}$$

- 2.3 Simplify the following:

- (a)  $4x + 2t + x - t$   
 (b)  $3 \times q \times q^2 \times s \times 2$   
 (c)  $\frac{14r}{21s}$

$$\begin{aligned} \text{(a)} \quad & \overset{\textcircled{1}}{4x} + \overset{\textcircled{2}}{2t} + \overset{\textcircled{3}}{x} - \overset{\textcircled{4}}{t} \quad \text{remember to add or subtract 'like' terms} \\ &= 5x + t \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 3 \times q \times q^2 \times s \times 2 \quad \text{remember when multiplying } p \times p = p^2 \\ &= 6q^3s \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \frac{2\cancel{14}r}{3\cancel{21}s} \\ &= \frac{2r}{3s} \end{aligned}$$

- 2.4 Expand and simplify the following:

- (a)  $4(p - 2)$   
 (b)  $-3x(x + 4p)$   
 (c)  $4(2a - 5) - 2(3a + 1)$

**Brackets mean multiply**

$$\begin{aligned} \text{(a)} \quad & \underbrace{4}_{\times} \underbrace{(p - 2)}_{\times} \\ &= 4 \times p - 4 \times 2 \\ &= 4p - 8 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \underbrace{-3x}_{\times} \underbrace{(x + 4p)}_{\times} \\ &= (-3x)(x) + (-3x)(4p) \\ &= -3x^2 - 12px \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & 4(2a - 5) - 2(3a + 1) \\ &= 8a - 20 - 6a - 2 \quad \text{collect 'like' terms} \\ &= 2a - 22 \end{aligned}$$

2.5 Expand and simplify:

- (a)  $(x + 4)(x + 3)$   
 (b)  $(2t - 1)^2$   
 (c)  $-2(a + 6)(a - 3)$

**Multiply the terms**

(a)  $(x + 4)(x + 3)$   $= (x \cdot x) + (x \cdot 3) + (4 \cdot x) + (4 \cdot 3)$   
 $= x^2 + 3x + 4x + 12$   
 $= x^2 + 7x + 12$

(b)  $(2t - 1)^2$   $= (2t - 1)(2t - 1)$   
 $= 4t^2 - 2t - 2t + 1$   
 $= 4t^2 - 4t + 1$

(c)  $-2(a + 6)(a - 3)$   $= -2(a^2 - 3a + 6a - 18)$   
 $= -2(a^2 + 3a - 18)$   
 $= -2a^2 - 6a + 36$

2.6 Factorise the following:

- (a)  $6 - 18p$   
 (b)  $-18x + 12y$   
 (c)  $-10ab - 20ac$

**Remove the highest common factor out the front of a bracket.**

(a)  $6 - 18p$   $= 6(1 - 3p)$   
*HCF is 6* *divide  $\frac{6}{6}$*  *divide  $\frac{18p}{6}$*

(b)  $-18x + 12y$   $= -6(3x - 2y)$   
*HCF is 6* *sign changes as a negative value is removed out the front of the bracket*

(c)  $-10ab - 20ac$   $= -10a(b + 2c)$   
*HCF is 10a* *be careful !!*

2.7 Solve each of the following **without** a calculator:

- (a)  $4p - 7 = 21$   
 (b)  $\frac{2a + 3}{5} = 3$   
 (c)  $\frac{n}{5} + 2 = -1$

**Remember**

- Variables to the left : numbers to the right.
- Change side : change sign.

(a)  $4p - 7 = 21$

$$4p = 21 + 7 \quad \text{add 7}$$

$$4p = 28$$

$$p = \frac{28}{4} \quad \text{divide by 4}$$

$$p = 7$$

(b)  $\frac{2a + 3}{5} = 3$

$$2a + 3 = 3 \times 5 \quad \text{multiply by 5}$$

$$2a + 3 = 15$$

$$2a = 15 - 3 \quad \text{subtract 3}$$

$$2a = 12$$

$$a = \frac{12}{2} \quad \text{divide by 2}$$

$$a = 6$$

(c)  $\frac{n}{5} + 2 = -1$

$$\frac{n}{5} = -1 - 2 \quad \text{subtract 2}$$

$$\frac{n}{5} = -3$$

$$n = -3 \times 5 \quad \text{multiply by 5}$$

$$n = -15$$

2.8 Solve the following **without** a calculator:

(a)  $6 - 2(4p + 5) = 10$

(b)  $\frac{5x + 2}{3} = \frac{2x - 3}{4}$

(c)  $\frac{7 - (g + 4)}{3} + 2 = 4(g - 6)$

(a)  $6 - 2(4p + 5) = 10$  *expand brackets*

$$6 - 8p - 10 = 10 \quad \text{collect like terms}$$

$$-8p - 4 = 10$$

$$-8p = 10 + 4$$

$$-8p = 14$$

$$p = -\frac{7}{4}$$

(b)  $\frac{5x + 2}{3} = \frac{2x - 3}{4}$  *cross multiply*

$$4(5x + 2) = 3(2x - 3)$$

$$20x + 8 = 6x - 9$$

$$20x - 6x = -9 - 8$$

$$14x = -17$$

$$x = -\frac{17}{14}$$

(c)  $\frac{7 - (g + 4)}{3} + 2 = 4(g - 6)$  *expand brackets*

$$\frac{7 - g - 4}{3} + 2 = 4g - 24$$

$$\frac{7 - g - 4}{3} = 4g - 24 - 2 \quad \text{collect like terms}$$

$$\frac{-g + 3}{3} = 4g - 26$$

$$-g + 3 = 3(4g - 26)$$

$$-g + 3 = 12g - 78$$

$$g + 12g = 78 + 3$$

$$13g = 81$$

$$g = \frac{81}{13}$$

2.9 Write an equation and solve it.

I think of a number, double it, add 5 and divide the answer by 3.  
If the result is 5, what is the number?

Let  $n$  be the number

$$\begin{array}{l} \curvearrowright \\ \text{double} \end{array} \frac{2n + 5}{3} = 5$$

$$2n + 5 = 15$$

$$2n = 10$$

$$n = 5$$

The number is 5

2.10 Solve the following inequalities:

(a)  $4x - 5 \leq 27$

(b)  $5(2x + 1) > 4(3x - 2)$

(a)  $4x - 5 \leq 27$

$$4x \leq 27 + 5$$

$$4x \leq 32$$

$$x \leq \frac{32}{4}$$

$$x \leq 8$$

(b)  $5(2x + 1) > 4(3x - 2)$  *expand brackets*

$$10x + 5 > 12x - 8$$

$$10x - 12x > -8 - 5$$

$$-2x > -13 \quad \text{when dividing by a negative value change the inequality sign}$$

$$x < \frac{-13}{-2}$$

$$x < \frac{13}{2}$$

2.11 Solve the following **without** a calculator.

(a)  $2x^2 = 32$

(b)  $5x^2 + 4 = 40$

(c)  $3^x = 27$

(d)  $8^{x+1} = 16$

(c)  $2x^2 = 32$

$$x^2 = \frac{32}{2}$$

$$x^2 = 16$$

$$x = \sqrt{16}$$

$$x = \pm 4 \quad \text{remember - two solutions}$$

$$\begin{aligned} \text{(d)} \quad 5x^2 + 4 &= 40 \\ 5x^2 &= 40 - 4 \\ 5x^2 &= 36 \\ x^2 &= \frac{36}{5} \\ x^2 &= \sqrt{\frac{36}{5}} \\ x &= \pm \frac{6}{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 3^x &= 27 \quad \text{change both values to the same base} \\ 3^x &= 3^3 \\ \therefore x &= 3 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 8^{x+1} &= 16 \\ (2^3)^{x+1} &= 2^4 \quad \text{multiply 3 by (x + 1)} \\ 2^{3x+3} &= 2^4 \\ \therefore 3x + 3 &= 4 \\ 3x &= 1 \\ x &= \frac{1}{3} \end{aligned}$$

## PROBLEMS TO SOLVE

### CHAPTER 1: ALGEBRAIC SKILLS

1. If  $m = -2$ ,  $n = -6$ ,  $p = 4$

evaluate

- (a)  $2mn - 3p$
- (b)  $m^2 + n^2 - p^2$
- (c)  $3 - (mnp)$
- (d)  $-2n - 3mp$
- (e)  $4m^2 - (3n + p - 6)$

2. The compound interest formula is  $A = P\left(1 + \frac{r}{100}\right)^t$  where

A = amount in the account

P = principal

r% = interest rate p.a.

t = time in years

Find the value in the account if:

- (a) \$4000 is deposited for 4 years at 6% p.a. compounded annually
- (b) \$12 000 is deposited for 6 years at 7.25% p.a. compounded annually

3. The formula for the surface area of a cylinder is:

$$S = 2\pi r^2 + 2\pi rh \text{ where}$$

r = radius

h = height

Calculate the surface area of a cylinder if

- (a) radius is 7 cm and height is 10 cm
- (b) radius is 10 cm and height is 0.2 m

4. Converting temperature in degrees Fahrenheit (F) to degrees Celsius (C) is given by:

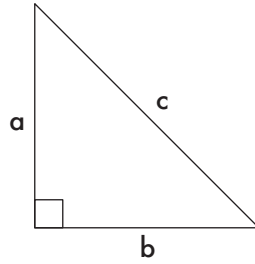
$$C = \frac{5}{9}(F - 32).$$

Convert the following to degrees Celsius:

- (a)  $42^\circ \text{ F}$
- (b)  $0^\circ \text{ F}$
- (c)  $100^\circ \text{ F}$

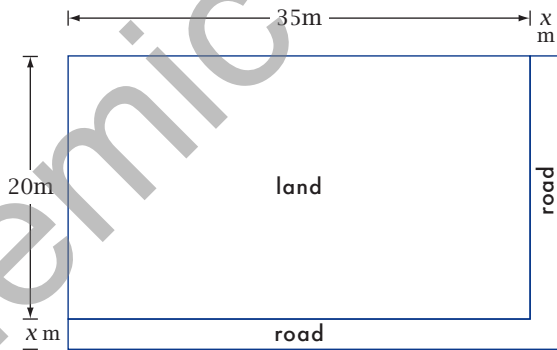
5. The formula to calculate the length of the hypotenuse on a right angled triangle is

$$c = \sqrt{a^2 + b^2}$$

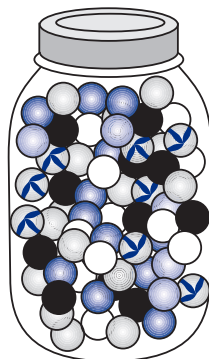


Calculate the length of the hypotenuse when:

- (a)  $a = 6 \text{ m}$ ,  $b = 8 \text{ m}$   
 (b)  $a = 2.5 \text{ cm}$ ,  $b = 4 \text{ cm}$
6. Write an algebraic expression for:
- (a) Four times the number  $p$  is subtracted from twelve.  
 (b) The number  $n$  is divided by three times the number  $t$ .  
 (c) The sum of six and the number  $f$  is multiplied by negative two.  
 (d) The difference between the number  $p$  and the number  $q$  is divided by the product of  $p$  and  $q$ .
7. For each of the following state an algebraic expression:
- (a) A rectangular block of land measures 35 metres by 20 metres. A road is constructed of width  $x$  metres as shown in the diagram below. Determine the total area of the land which includes the road.



- (b) Sam buys a container with  $m$  marbles.
- i. If she equally divides them amongst  $p$  people how many marbles will each person receive?
  - ii. Sam decides to keep  $\frac{1}{3}$  of the marbles and divides the remaining ones amongst  $t$  people. How many marbles will each person receive?





7. (c) For an 8 hour work day Sydney uses the car for ' $h$ ' hours. Her brother Tom uses it for  $\frac{1}{4}$  of the remaining time. Determine how long Sydney and Tom use the car over an 8 hour day.
8. Simplify the following algebraic expressions:
- $m + m + m - n - n$
  - $t + 6t$
  - $5m^2 - m^2$
  - $4x^2 - 2x + 5 - 6x - 8 + 3x^2$
9. Simplify the following:
- $-3a \times -5ab \times -10abc$
  - $(-3xy)^2$
  - $\frac{6a^2b}{-12a}$
  - $\frac{-25m}{-35n}$
10. If  $a = 6$  and  $b = -2$  find the value of:
- $ab$
  - $3a - 2b$
  - $a - b^2$
11. Expand the following:
- $5(y - 6)$
  - $-2x(4 - 3f)$
  - $3h(2h - 5g)$
  - $-(3p + 2q)$
12. Expand and simplify each of the following:
- $7(2a - 5) - 4a$
  - $3(2p - 1) + 2(p + 6)$
  - $5(3e - 2) - 2(4 - e)$
  - $-3x(2x - 3) - 2x(5x + 1)$
13. Expand and simplify the following:
- $(p - 2)(p - 9)$
  - $-3(2y - 7)^2$
  - $(8w - 3)(2w - 3)$
  - $-2(5c - 4)(3c - 2)$
14. Factorise the following:
- $7 - 21b$
  - $-16 - 12y$
  - $15pq - 25ps$
  - $x^2 - 10x$

15. Solve each of the following equations **without** a calculator:

(a)  $x - 6 = 10$

(b)  $2y = 12$

(c)  $\frac{p}{7} = -3$

(d)  $9 - t = 5$

16. Solve each of the following equations **without** a calculator:

(a)  $-9x + 5 = 86$

(b)  $\frac{3p - 2}{5} = 8$

(c)  $\frac{6}{x} = 5$

(d)  $\frac{3}{4}b + 5 = -2$

17. Solve each of the following **without** a calculator:

(a)  $x - 9 = 12 - 2x$

(b)  $2(z + 4) + 4z = -32$

(c)  $-3(4m - 5) + 6m = 9$

(d)  $\frac{15 - b}{2} + 3 = 12$

18. Solve the following **without** a calculator:

(a)  $7(2x + 3) - 6(3x - 1) = -1$

(b)  $\frac{3x - 7}{2} = x + 5$

(c)  $\frac{4x + 5}{3} = \frac{2x + 7}{2}$

(d)  $5g - 2 = 8 - (10 - g)$

19. Write an equation for each of the following and then solve it:

(a) I think of a number, multiply it by three, add five and the result is negative ten.

(b) Three is subtracted from one quarter of a number and the result is seven.

(c) Twice a number is subtracted from eight and the answer divided by seven giving a result of four.

(d) A man travels 'k' kilometres on the first day. On each of the next two days he travels twice as far as he did on the first day. On each of the two remaining days he travels 10 more kilometres than what he did on the second day. If the total journey is 524km, how far did he travel on the first day?

20. Solve each of the following inequalities **without** a calculator:

(a)  $8 + 3x \geq -13$

(b)  $2x - 1 < 5x + 7$

(c)  $6(3x - 1) \leq 5(4x - 3)$

(d)  $\frac{5 - 2x}{6} > -7$

21. Solve the following equations **without** the use of a calculator:

(a)  $9x^2 = 25$

(b)  $x^2 - \frac{1}{25} = 0$

(c)  $6(p^2 - 1) = 37$

(d)  $-2t^3 + 4 = 132$

22. Solve the following equations **without** a calculator:

(a)  $3^{x+1} = 27$

(b)  $7^x = \frac{1}{49}$

(c)  $\frac{5^x}{5} = 125$

(d)  $10^x = 0.0001$

23. Solve the following equations using a calculator correct to 2 decimal places.

(a)  $2^x = 7$

(b)  $3^{x+1} = 15$

(c)  $7^x = \frac{1}{25}$

(d)  $\frac{4^{x-2}}{3} + 2 = 11$